Exercise 9

Suppose $y = \sqrt{2x+1}$, where x and y are functions of t.

(a) If $dx/dt = 3$, find dy/dt when $x = 4$.

(b) If $dy/dt = 5$, find dx/dt when $x = 12$.

Solution

Find the derivative of y with respect to t by using the chain rule.

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt}\sqrt{2x+1} \\ &= \frac{1}{2}(2x+1)^{-1/2} \cdot \frac{d}{dt}(2x+1) \\ &= \frac{1}{2}(2x+1)^{-1/2} \cdot \left(2\frac{dx}{dt}\right) \\ &= \frac{1}{\sqrt{2x+1}}\frac{dx}{dt}\end{aligned}$$

Part (a)

When $dx/dt = 3$ and $x = 4$,

$$\left.\frac{dy}{dt}\right|_{x=4} = \frac{1}{\sqrt{2(4)+1}}(3) = 1.$$

Part (b)

Solve the equation for dx/dt by multiplying both sides by $\sqrt{2x+1}$.

$$\frac{dx}{dt} = \frac{dy}{dt}\sqrt{2x+1}$$

When $dy/dt = 5$ and $x = 12$,

$$\left.\frac{dx}{dt}\right|_{x=12} = (5)\sqrt{2(12)+1} = 25.$$